LESSON 7:

Discriminant and Nature of Roots of Quadratic Equations

We use the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for the roots of the quadratic equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$.

The expression under the radical sign of the quadratic formula plays an important role in the calculation of the roots. This expression enables us to determine the discriminant and nature of roots without solving the equation. By the nature of roots we mean:

- **whether the equation has real roots.**
- **if there are real roots, whether they are different or equal.**

The expression **$b^2 - 4ac$** is called the **discriminant** of the quadratic equation because it discriminates among the four cases which can occur.

> If **$b^2 - 4ac = 0$**, then the roots are **real, rational, equal**.
> If **$b^2 - 4ac > 0$** and is a perfect square, then the roots are
> **real, rational, unequal**.
> If **$b^2 - 4ac > 0$** and is not a perfect square, then the roots are
> **real, irrational, unequal**.
> If **$b^2 - 4ac < 0$**, then the roots are **non-real, imaginary**.

Example: Determine the discriminant and nature of roots of each quadratic equation.
(a) $x^2 - 6x + 9 = 0$ (c) $x^2 - 7x - 4 = 0$
(b) $x^2 - 4x + 3 = 0$ (d) $2x^2 + 3x + 5 = 0$

Solution: (a) $x^2 - 6x + 9 = 0$ $a = 1;\ b = -6;\ c = 9$
$\qquad D = b^2 - 4ac$
$\qquad\quad = (-6)^2 - 4(1)(9)$
$\qquad\quad = 36 - 36$
$\qquad$ **D = 0**;   **real, rational, equal**

(b) $x^2 - 4x + 3 = 0$ a = 1; b = -4; c = 3

$D = b^2 - 4ac$

$= (-4)^2 - 4(1)(3)$

$= 16 - 12$

D = 4;   **real, rational, unequal**

(c) $x^2 - 7x - 4 = 0$ a = 1; b = -7; c = -4

$D = b^2 - 4ac$

$= (-7)^2 - 4(1)(4)$

$= 49 + 16$

D = 65;   **real, irrational, unequal**

(d) $2x^2 + 3x + 5 = 0$ a = 2; b = 3; c = 5

$D = b^2 - 4ac$

$= (3)^2 - 4(2)(5)$

$= 9 - 40$

D = -31; **non – real, imaginary**

Problem Set No. 7

A. Describe the nature of roots of a quadratic equation given the value of the discriminant. Write your answer on the space provided.

_____ 1. 36   _____ 6. -49

_____ 2. -17   _____ 7. 676

_____ 3. 0   _____ 8. -100

_____ 4. 196   _____ 9. 1

_____ 5. 143   _____ 10. 2025

B. Find the value of the discriminant and nature of zeros in each quadratic function.

$ax^2 + bx + c = 0$	Discriminant	Nature of Roots
1. $x^2 - 2x + 4 = 0$		
2. $x^2 + 4x - 21 = 0$		
3. $x^2 + 3x + 3 = 0$		
4. $x^2 - 5x + 12 = 0$		
5. $x^2 - 9x + 7 = 0$		